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ENERGY FLOW IN NETWORKS

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ABSTRACT

All networks are formed by the use of dissipators and storage elements. In addition, in order to couple one network with another, transformers and transducers are utilized. These facts are well known in the electrical field, but not so well known in the other disciplines.

The purpose of this general paper is to show how some common devices may be coupled one to the other to form energy networks. The mathematics of coupling follows the work of Oskar Lange in which a general mode of mathematical description for all networks is developed, through the use of matrix algebra.

Two simple examples are developed in the paper which are soluble without computer assistance. However, the greatest advantage of the technique is its compactness of notation and the ability to extend the technique to large and complex networks.

NOMENCLATURE

F	generalized through variable
P	generalized across variable
q	heat flow
T	torque
	force
v, V	velocity
Q	volumetric flow rate
Δp	pressure drop
M, m	mass
ΔT	temperature difference
r	gear radius
N	number of gear teeth
C_p	specific heat
A	cross-sectional area
ΔH_v	heat of vaporization
$\bar{T}$	connectivity matrix
a	element in $\bar{T}$

$\bar{I}_c$	unit column matrix
j	entity stored (mass, heat, etc.)
E	energy (kinetic, potential, enthalpy)
h	enthalpy
g_c	constant
$\dot{Q}$	heat flux column vector
$\dot{W}$	work flux (power) column vector
Δj_c	energy storage column vector
q_b	heat to boiler
q_c	heat out at condenser
q_{pl}	heat lost in pipelines
W_T	work out at turbine
Ω	angular velocity

Subscripts

v	vapor
p	constant pressure

Superscripts

c	condenser
W_{pu}	work in at pump
Q_{Tc}	total energy for cycle

TRANSFORMERS AND TRANSDUCERS

Transformers (energy–energy transformers)

The network transformer thought of most frequently is the electrical transformer shown schematically in Fig. 1; this device couples two electrical networks together so that energy may be transferred.

A number of other forms are known (see Fig. 2) in systems theory[2] such as the simple lever (couples two mechanical networks), gears (couples two mechanical networks) and fluid transformers (couples two fluid networks). However, it is possible to look at heat exchangers as thermal transformers, as shown in Fig. 3. More importantly, the function and mathematical models of transformers match that of the heat exchanger. In Table 1 the mathematical models for a few transforming devices

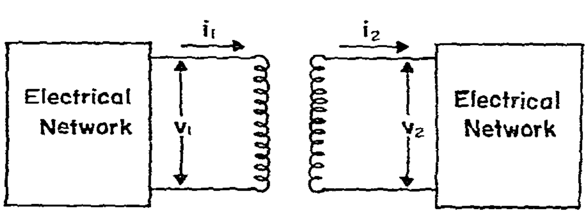

Fig. 1. An electrical transformer coupling two electrical networks.

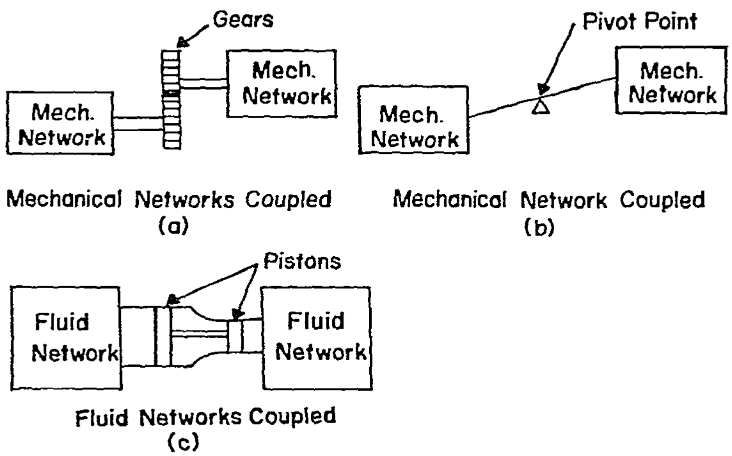

Fig. 2. Transformers coupling networks.

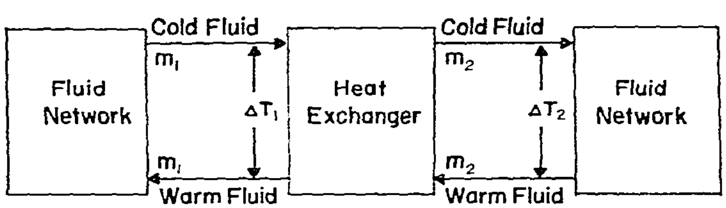

Fig. 3. A thermal transformer.

compared to the heat exchanger are shown. A name for a heat exchanger might now be a heat energy or thermal transformer. The usual definition of a pure transformer is that it is a device which changes the level of output variables relative to input variables yet sustains no power loss in the transformation. This may be written from a power balance generally as
Power in = Power out

$$F_1P_1 = F_2P_2 \tag{1}$$

F represents a generalized variable (the flow in system); P represents a generalized cross variable (potential or driving force in system); 1, 2 input and output at terminals, respectively (ground potential not shown).

Note that in Table 1 the simple power balance in (1) is used assuming no losses of storage through the units. The only unfamiliar case is that of the thermal transformer, since

$$q_1 = q_2 \tag{2}$$

This assumes no heat loss in the unit which is similar to the statement that power is not lost in the transfer of power through the usual transformers. There

$$C_{p1}m_1\Delta T_1 = C_{p1}M_2\Delta T_2$$

or

$$C_{t1}\Delta T_1 = C_{t2}\Delta T_2$$

TABLE 1

SIMPLE TRANSFORMER MATHEMATICAL MODELS

	Gears	Levers	Electrical	Fluid	Thermal
Power transferred	$T_1\Omega_1 = T_2\Omega_2$	$f_1v_1 = f_2v_2$	$i_1v_1 = i_2v_2$	$Q_1\Delta P_1 = Q_2\Delta P_2$	$q = C_pM\Delta T$ $q_1 = q_2$
System ratio	$\frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{N_1}{N_2} = -\frac{1}{n}$	$\frac{F_1}{F_2} = \frac{r_2}{r_1} = -\frac{1}{n}$	$\frac{i_1}{i_2} = -\frac{v_2}{v_1} = \frac{N_2}{N_1} = -\frac{1}{n}$	$\frac{Q_1}{Q_2} = \frac{v_1}{v_2} = -\frac{1}{n}$	$\frac{\Delta T_1}{\Delta T_2} = \frac{C_{t_2}}{C_{t_1}} = n$
	$\frac{\Omega_1}{\Omega_2} = -\frac{r_2}{r_1} = -\frac{N_2}{N_1} = n$	$\frac{v_1}{v_2} = \frac{r_1}{r_2} = n$	$\frac{v_1}{v_2} = \frac{N_1}{N_2} = n$	$\frac{\Delta P_1}{\Delta P_2} = \frac{A_2}{A_1} = n$	

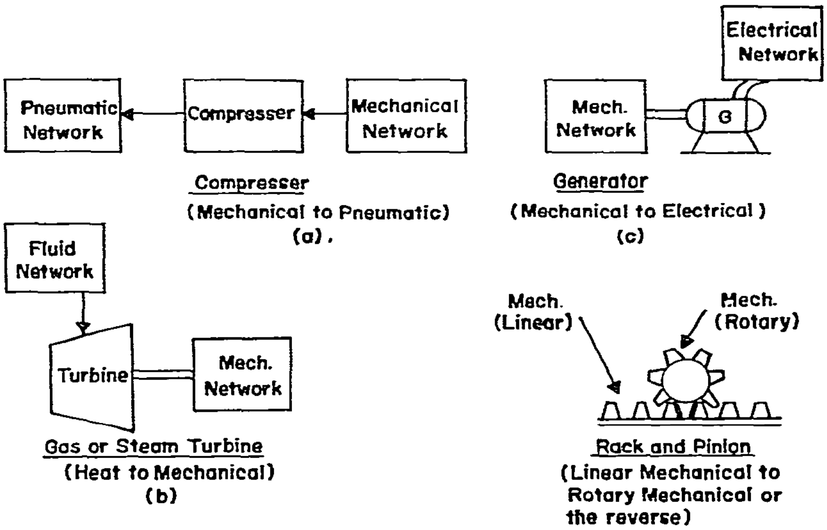

Fig. 4. Transducers coupling networks.

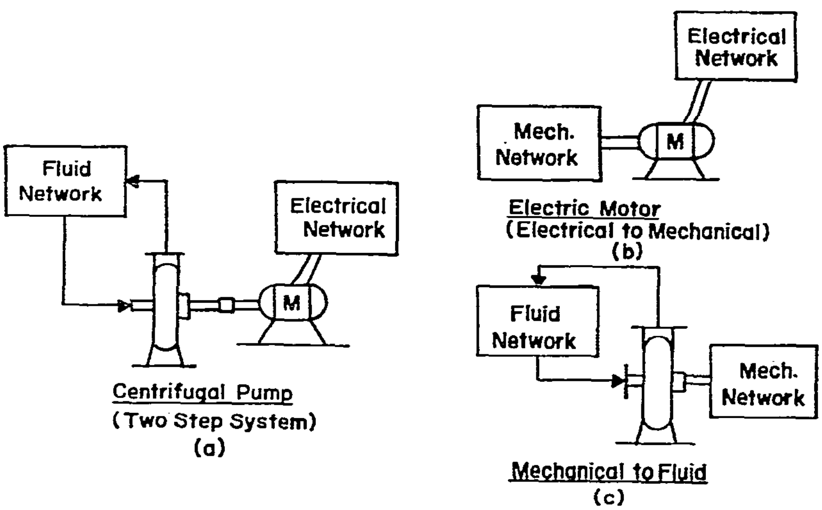

Fig. 5. Transducers coupling networks.

$$\frac{\Delta T_1}{\Delta T_2} = \frac{C_{t_2}}{C_{t_1}} \quad (3)$$

This equation is quite similar to the other usual forms of relative potentials to some geometrical aspect of the system (gear diameter, fulcrum distances from effort or load, turns in electrical transformer and area of fluid transformer pistons).

Transducers

Energy–energy. Devices for the transduction of energy are also quite common and are shown in Fig. 4. Note here again that transducers couple networks so that energy may flow between networks. A two-step transduction has been broken into its components in order to indicate transducer coupling. Other usual simple transducers are shown in Fig. 5.

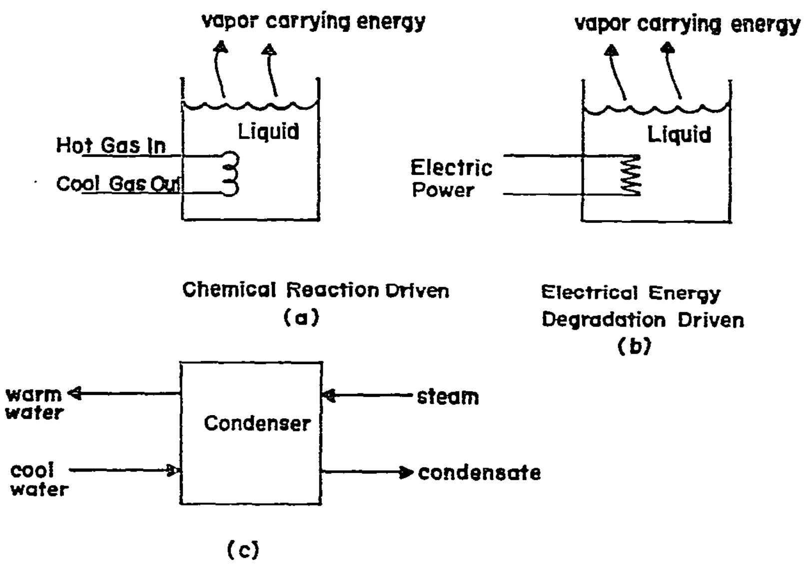

Fig. 6. Energy–matter transducers.

Recall that the definition of a transducer is that it is device which transforms energy or power in one form into energy or power in another form. If no energy losses are permitted then generally

$$F_1P_1 = F_2P_2 \tag{4}$$

This equation is identical to the one used for transformers.

Energy–matter or energy–energy. It is possible to think of boilers, tea kettles, etc. as both energy–matter and energy–energy transducers (see Fig. 6). A boiler being fired by hot gases produces steam (which carries energy) in proportion to the heat transferred to the water. If one puts a boiler (or tea kettle) in a black box with energy going into one pipe and steam issuing through another pipe then all we would know is that when more energy is added into the box more steam issues. One may then state that a phase transition mass transfer device driven by energy (or the removal of it) is an energy–matter transducer. A steam condenser is the reverse type of transducer — the more steam condensed the more heat that must be carried away with the cooling water.

In these types of transducers a distinct bi-functionality exists in the transported matter; the matter (steam) carries energy and both increase in outflow from a boiler boiler when energy is added.

Simple boiler equations

The mathematical model for this transduction step (assuming no heat losses) is

$$Q_1 = m_2 \Delta H_v \tag{5}$$

Q_1 = heat flux into boiler water from chemical reaction or electrical degradation step, heat/unit time; m_2 = mass of steam emitted from boiler/unit time; ΔH_v = heat of vaporization of steam, heat/unit mass; or

$$\frac{Q_1}{\Delta H_v} = m_2 \tag{6}$$

which relates now the net heat flux into the boiler system to mass flow rate (mass flux). ΔH_v is assumed constant here.

Simple condenser equation

The mathematical model which relates steam (vapor) condensed in a heat exchanger to heat absorption is shown below assuming no heat losses through condenser shell

$$Q_1^c = m_2^c \Delta H_v \tag{7}$$

Q_1^c = heat absorbed by exchanger through water cooling system; m_2^c = mass of steam or vapor condensed in exchanger per unit time; ΔH_v = heat of vaporization or condensation in heat/unit mass.

If as an approximation losses through the shell are neglected, then

$$\frac{Q_1^c}{\Delta H_v} = m_2^c \tag{8}$$

The greater the heat absorption by the cooling water the higher the mass of vapor condensed. This is defined as a transduction step.

Transducers and transformers can be connected into complex configurations in order to accomplish some purpose. These connections may be shown quite simply[4] in matrix form. The remainder of this paper will indicate how these connections can be shown and two simple examples of its use will be given.

CONNECTIVITY AND THE FLOW OF ENTITY

The Kirchhoff matrix

It is possible to assemble elements, units, or nodes into a network which has as its branches flows of some entity such as charge, energy, fluid, people, products, etc. All networks formed by man or nature[3] have a rational basis for existence. Figure 7 shows a three-element network so connected that all elements interact with all other elements and with themselves. The connectivity may be put into a matrix form as

$$\overline{T} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{9}$$

However, not only are the elements signalled by subscripts to indicate con-

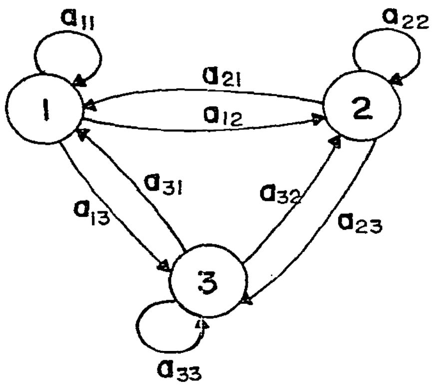

Fig. 7. A three-element network.

nections but the elements themselves may indicate the transport of mass and energy (or power), i.e.

a_{11}, a_{22}, a_{33} feed back flow of some entity to elements;
a_{12}, a_{13} flow out of some entity element (1);
a_{21}, a_{23} flow out of some entity element (2);
a_{31}, a_{32} flow out of some entity element (3).

These elements as written above can constitute the rows in the square matrix, therefore elements of node outputs are formed in the rows of the connection-transport matrix. The columns then indicate the inputs into elements or nodes as

a_{11}, a_{21}, a_{31} inputs into element (1);
a_{12}, a_{22}, a_{32} inputs into element (2);
a_{13}, a_{23}, a_{33} inputs into element (3).

This simple matrix then indicates all input and output flows of entity for a given system (network). If we could relate them a Kirchhoff form of expression might be developed. This is easily done by obtaining the transposition of (10) and subtracting it from (10) as

$$\bar{T} - \bar{T}^* = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} - \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \tag{10}$$

or

$$\bar{T} - \bar{T}^* = \begin{bmatrix} (a_{11} - a_{11}) & (a_{12} - a_{21}) & (a_{13} - a_{31}) \\ (a_{21} - a_{12}) & (a_{22} - a_{22}) & (a_{23} - a_{32}) \\ (a_{31} - a_{13}) & (a_{32} - a_{23}) & (a_{33} - a_{33}) \end{bmatrix} \tag{11}$$

At this point it is noticed that what has been done is to evaluate the net flow out of each element. The principal diagonal terms cancel leaving the interaction (off-diagonal) elements. If the matrix in (12) is post-multiplied by a column matrix then a row summation is achieved.

$$(\bar{T} - \bar{T}^*) = \begin{bmatrix} 0 + a_{12} - a_{21} + a_{13} - a_{31} \\ a_{21} - a_{12} \quad 0 \quad a_{23} - a_{32} \\ a_{31} - a_{13} + a_{32} - a_{23} \quad 0 \end{bmatrix} \tag{12}$$

The simple Kirchhoff expression for the system is

$$
\begin{aligned}
0 + a_{12} - a_{21} + a_{13} - a_{31} &= 0 \\
a_{21} - a_{12} + 0 + a_{23} - a_{32} &= 0 \\
a_{31} - a_{13} + a_{32} - a_{23} + 0 &= 0
\end{aligned} \tag{13}
$$

So that

$$
\begin{aligned}
a_{12} + a_{13} - a_{21} - a_{31} &= 0 \\
a_{21} + a_{23} - a_{12} - a_{32} &= 0 \\
a_{31} + a_{32} - a_{13} - a_{23} &= 0
\end{aligned} \tag{14}
$$

or generally in matrix form

$$(\bar{T} - \bar{T}^*)\bar{I}_c = \bar{0} \tag{15}$$

This Kirchhoff matrix expression is valid for the flow of any entity in any network. In any practical problem not all branches are operative which means that not all the a's are utilized. In the three-element problem only three flows can be unknown; or generally for an n-noded network only n number of entity flows can be unknown.

Continuity of entity

Expression (16) as developed assumes no accumulation or depletion in the elements. This facet of the analysis is necessary since many systems do store entity (charge, mass, people, energy). Let j_i = amount of entity stored initially ($t = 0$); j_f = amount of entity stored finally ($t = t$); Δj = amount of entity accumulated or depleted in time t. The continuity equation for networks may be written as

$$\begin{Bmatrix} \text{net flow} \\ \text{out of entity} \end{Bmatrix} + \begin{Bmatrix} \text{accumulation} \\ \text{or} \\ \text{depletion of entity} \end{Bmatrix} = 0 \tag{16}$$

Therefore using Δj in matrix form

$$(\bar{T} - \bar{T}^*)\bar{I}_c + \Delta j = \bar{0} \tag{17}$$

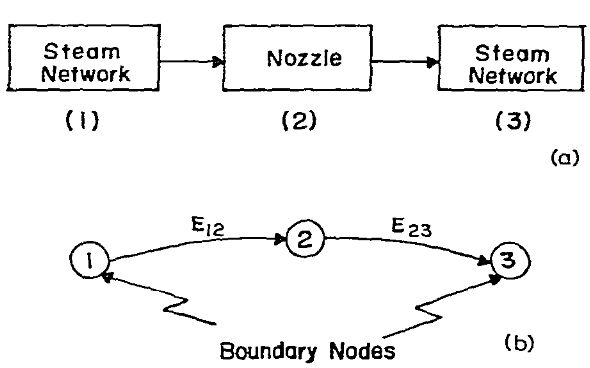

Fig. 8. Schematic (a) and network diagrams (b) for steam nozzle.

Equation (17) can be used to evaluate depletion, accumulation and network branch flows in large, complex networks[4].

Example: the steam nozzle. An example of the use of the continuity matrix expression is indicated below. Consider a steam nozzle; this might couple two steam piping networks which perform some processing function. It is proposed here to study the system from a network basis (see Fig. 8). This simple problem is instructive since it clearly indicates the absorption and rejection of energy (enthalpic and kinetic energy) out of and into storage elements or nodes. If no heat loss to the environment exists through the nozzle body or lines connecting the reservoirs then the problem is quite simple. The connection-transport matrix is

$$\bar{T} = \begin{bmatrix} 0 & E_{12} & 0 \\ 0 & 0 & E_{23} \\ 0 & 0 & 0 \end{bmatrix} \tag{18}$$

The transpose of (18) is

$$\bar{T}^* = \begin{bmatrix} 0 & 0 & 0 \\ E_{12} & 0 & 0 \\ 0 & E_{23} & 0 \end{bmatrix} \tag{19}$$

Assuming no storage of energy in the nozzle (2) then

$$(\bar{T} - \bar{T}^*)\ \bar{I}_c + \Delta j = \begin{bmatrix} E_{12} \\ E_{23} - E_{12} \\ -E_{23} \end{bmatrix} + \begin{bmatrix} \Delta j_1 \\ \Delta 0 \\ \Delta j_3 \end{bmatrix} = 0 \tag{20}$$

So that

$$\begin{aligned} E_{12} &= -\Delta j_1 \\ E_{23} - E_{12} &= 0 \\ -E_{23} &= -\Delta j_3 \end{aligned} \tag{21}$$

Equations (21) indicate what is happening. The source of energy is depleting at an E_{12} rate*; the sink of energy is filling (accumulating) at an E_{23} rate.

Since no storage is occurring at (2) then $E_{23} - E_{12} = 0$. The usual expression is shown as

$$h_1 + \frac{V_1^2}{2g_c} - h_2 + \frac{V_2^2}{2g_c} = 0 \tag{22}$$

The source steam network is losing energy at a E_{12} or $h_1 + V_1^2/2g_c$ rate and the sink steam network is gaining energy at a rate of $h_2 + V_1^2/2g_c$. Note that usually flowing steam velocity and pressures are changed as they have through the nozzle — this is a transforming step.

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* E_{12} is usually an energy/unit mass. If this is multiplied by the mass rate or flow then energy rate or power is obtained.

Another important aspect of networks is shown in this example; the boundary nodes (1) and (3) are apparent since they either have inputs or outputs, but not both.

Overall system balances (energy, matter, charge) can be made by studying the boundary nodes inputs and outputs.

Energy network vectors (first law of thermodynamics). Equation (23) is shown below as the usual energy equation in vector form. It is then possible to speak of thermodynamic networks as having heat vectors, work vectors, energy flow vectors and energy storage vectors*.

Heat vector		Work vector		Energy flow vector		Energy storage vector	
$\dot{Q}$	$-$	$\overline{W}$	$=$	$\bar{D}_{mn}\bar{I}_c$	$+$	$\overline{\Delta j_c}$	(23)

The matrix $\bar{D}_{mn}$ is composed of row (or column) vectors which are each related to the flows into and out of all nodes. Also the elements in this matrix are not simple and they themselves are the scalars resulting from the multiplication of energy and unit vectors.

Since generally it is now shown that $\bar{D}_{mn} = \bar{T} - \bar{T}^*$, then for energy (note eqn. 17), $D_{mn} = \bar{E} - \bar{E}^*$

$$\bar{D}_{mn} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} \cdots E_{1n} \\ E_{21} & E_{22} & E_{23} & \\ E_{31} \text{ etc.} & & & \\ E_{41} & & & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ E_{n1} & & & \end{bmatrix} - \begin{bmatrix} E_{11} & E_{21} & E_{31} & \cdot\cdot E_{n1} \\ E_{12} & E_{22} & E_{23} & \\ E_{13} \text{ etc.} & & & \\ E_{14} & & & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ E_{1n} & & & \end{bmatrix} \tag{24}$$

But actually

$$E_{11} = h_{11} + \frac{V_{11}^2}{2g_c} + Z_{11}\frac{g}{g_c} \tag{25}$$

$$E_{12} = h_{12} + \frac{V_{12}^2}{2g_c} + Z_{12}\frac{g}{g_c}$$

so that

$$E_{11} = \left(h_{11}, \frac{V_{11}^2}{2g_c}, Z_{11}\frac{g}{g_c}\right) \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$E_{12} = \left(h_{12}, \frac{V_{11}^2}{2g_c}, Z_{12}\frac{g}{g_c}\right) \begin{bmatrix}1\\1\\1\end{bmatrix} \tag{26}$$

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* Heat and work could be defined as storage vectors; it has not been done here in order to hold similarity to the first law forms.

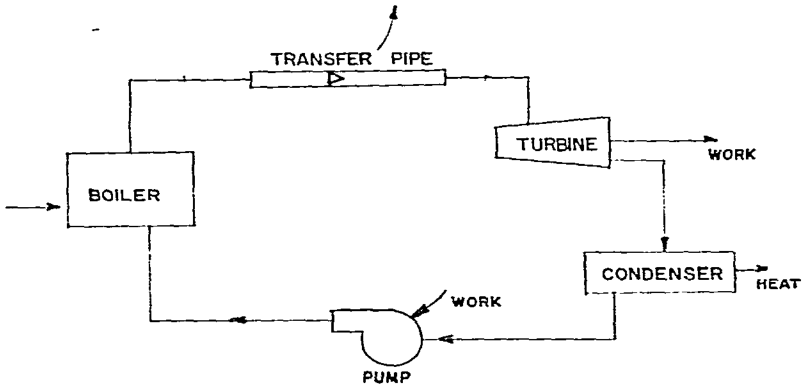

Fig. 9. Steam power plant (schematic).

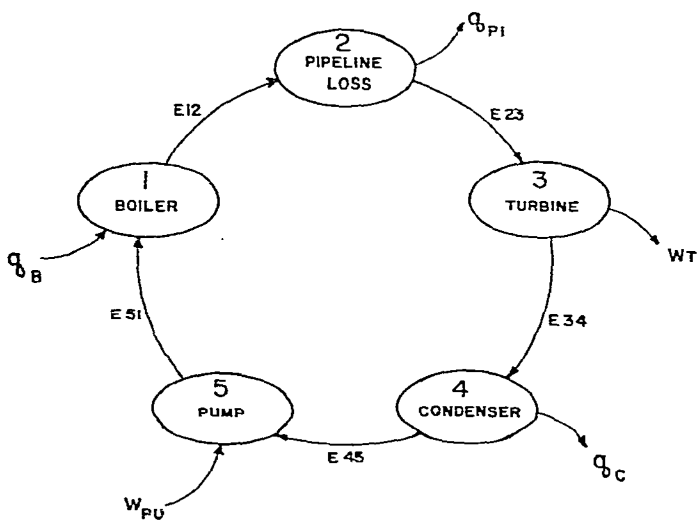

Fig. 10. Steam power plant network.

Example of use of matrix method: simple power plant network. Energy matrix. An example of the application of this network technique is indicated by using the steam power example given by Van Wylen and Sonntag[5]. It is instructive to compare the approach by these authors and the use of a generalized network mathematics.

The diagram of the steam power plant is indicated in Fig. 9. Figure 10 is a network representation of the same plant. Note that the plant is simple with a quite obvious cyclic nature.

The transforming and transducing elements in this power system are

Unit	*Function*	*Type*
Boiler	Energy–matter	Transducer
Pipeline loss	Matter–energy	Transducer
Turbine	Energy–energy	Transducer
Condenser	Matter–energy	Transducer
(Can also be used in the form of a transformer)		
Pump	Energy–energy	Transducer

Imagine, in each of the above cases, that you placed the unit in a black box and operated the unit. Noting what happens to changes in matter and energy variables, as one or the other is modified, gives one a clear idea of energy flows.

For the system shown in Fig. 9, find

(a) the heat transfer in line between boiler and turbine, q_{p1};

(b) turbine work, W_T;

(c) heat transferred in condenser, q_c;

(d) heat transferred in boiler, q_b.

from the given data (from Van Wylen and Sonntag[5]):

E_{12} = 1315 BTU)lb E_{34} = 1045 BTU/lb E_{51} = ?
E_{23} = 1289 BTU/lb E_{45} = 78 BTU/lb W_{pu} = 3 BTU/lb

Applying eqn. (23) to this example gives

$$\begin{bmatrix} +q_b \\ +q_{p1} \\ 0 \\ +q_c \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ W_T \\ 0 \\ +W_{pU} \end{bmatrix} = \left\{ \begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} & E_{15} \\ E_{21} & E_{22} & E_{23} & E_{24} & E_{25} \\ E_{31} & E_{32} & E_{33} & E_{34} & E_{35} \\ E_{41} & E_{42} & E_{43} & E_{44} & E_{45} \\ E_{51} & E_{52} & E_{53} & E_{54} & E_{55} \end{bmatrix} - \begin{bmatrix} E_{11} & E_{21} & E_{31} & E_{41} & E_{51} \\ E_{12} & E_{22} & E_{32} & E_{42} & E_{52} \\ E_{13} & E_{23} & E_{33} & E_{43} & E_{53} \\ E_{14} & E_{24} & E_{34} & E_{44} & E_{54} \\ E_{15} & E_{25} & E_{35} & E_{45} & E_{55} \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (27)$$

This is the energy form of (23) — not power.

Since there are no feedbacks on the individual equipment nodes a number of elements in the energy matrix are zero. Note also self-looping is zero (E_{11}, E_{22}, etc. = 0).

$$\begin{bmatrix} +q_b \\ +q_{p1} \\ 0 \\ +q_c \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ W_T \\ 0 \\ +W_{pU} \end{bmatrix} = \left\{ \begin{bmatrix} 0 & E_{12} & 0 & 0 & 0 \\ 0 & 0 & E_{23} & 0 & 0 \\ 0 & 0 & 0 & E_{34} & 0 \\ 0 & 0 & 0 & 0 & E_{45} \\ E_{51} & 0 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & E_{51} \\ E_{12} & 0 & 0 & 0 & 0 \\ 0 & E_{23} & 0 & 0 & 0 \\ 0 & 0 & E_{34} & 0 & 0 \\ 0 & 0 & 0 & E_{45} & 0 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (28)$$

Expression (42) may be written as

$$\begin{aligned} +q_B &= E_{12} - E_{51} \\ +q_{p1} &= E_{23} - E_{12} \\ -W_T &= E_{34} - E_{23} \\ +q_c &= E_{45} - E_{34} \\ -W_{pU} &= E_{51} - E_{45} \end{aligned} \quad (29)$$

Total energy requirement for plant

The total net energy requirement for the cycle is obtained by considering the inputs and outputs; those considered would be only those items from or into the environment.

For the cycle

$$E_{Tc} = (+q_B, +q_{p1}, 0, +q_c, 0)\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} + (0, 0, +W_T, 0, +W_{pU})\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} \tag{30}$$

$$E_{Tc} = (q_B + q_{p1} + q_c) + (W_T + W_{pU})$$

For a true cycle $E_{Tc} = 0$ and therefore

$$(q_B + q_{p1} + q_c) + (W_T + W_{pU}) = 0 \tag{31}$$

Numerical solution

See Van Wylen and Sonntag[5] for their detailed assumptions and solutions).

(1) No heat is lost or gained throughout the system aside from those stated.

(2) No kinetic energy and potential energy changes.

(3) No heat storage, $\Delta j = 0$.

Energy equation (see eqn. (28) or (29)

$$\begin{bmatrix}+q_B\\+q_{p1}\\0\\+q_c\\0\end{bmatrix} - \begin{bmatrix}0\\0\\W_T\\0\\-3\end{bmatrix} = \begin{bmatrix}0 & 1315 & 0 & 0 & 0\\0 & 0 & 1289 & 0 & 0\\0 & 0 & 0 & 1045 & 0\\0 & 0 & 0 & 0 & 78\\E_{51} & 0 & 0 & 0 & 0\end{bmatrix} - \begin{bmatrix}0 & 0 & 0 & 0 & E_{51}\\1315 & 0 & 0 & 0 & 0\\0 & 1289 & 0 & 0 & 0\\0 & 0 & 1045 & 0 & 0\\0 & 0 & 0 & 78 & 0\end{bmatrix} \tag{32}$$

$$\begin{aligned}q_B &= 1315 - E_{51}\\q_{p1} &= 1289 - 1315\\-W_T &= 1045 - 1289\\q_c &= 78 - 1045\\+3 &= E_{51} - 78\end{aligned} \tag{33}$$

$$\begin{aligned}\text{(wanted) } q_B &= 1315 - E_{51}\\\text{(wanted) } q_{p1} &= -26 \text{ Btu/lb}\\\text{(wanted) } W_T &= +244 \text{ Btu/lb}\\\text{(wanted) } q_c &= -967 \text{ Btu/lb}\\E_{51} &= +81 \text{ Btu/lb}\\q_B &= 1315 - 81 = +1234 \text{ Btu/lb}\end{aligned} \tag{34}$$

Consequently the overall boundary node energy balance (see eqn. (30))

$$E_{Tc} = (q_B, q_{p1}, 0, q_c, 0)\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} + (0, 0, W_T, 0, W_{pU})\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$$

$$E_{\mathrm{Tc}} = (1234,\ -26, 0,\ -967, 0)\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} + (0, 0,\ +244, 0,\ -3)\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$$

$E_{\mathrm{Tc}} = (1234\ -26\ -967) + (244\ -3)$
$E_{\mathrm{Tc}} = 241\ -241$
$E_{\mathrm{Tc}} = 0$ Btu/lb
Since $E_{\mathrm{Tc}} = 0$ the cycle is a true thermodynamic cycle.

CONCLUSIONS

(1) The usual energy converters are shown in the paper as transducers.

(2) A simple method is indicated which forms a connection matrix for transducer and transformer networks.

(3) Using the first law of thermodynamics it is possible to cast that law into a scalar matrix expression for computation use.

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